A Glossary for Pre-calculus

Absolute value (also known as modulus) of a complex number: If a complex number z = a + bi is represented as a vector from (0, 0) to (a, b) in a plane, the absolute value |z| is the length of that vector. $|z| = \sqrt{a^2 + b^2}$ |z| is also the distance of the point (a, b) from the origin.

Argument of a complex number: If a complex number z = a + bi is represented as a vector from (0,0) to (a, b) in a plane, the argument α of z is the counterclockwise angle between the positive x-axis and the vector. In the first quadrant, $\alpha = \tan^{-1}(b/a) = \arctan(b/a)$. See trigonometric form of a complex number.

Argument of a function: The input to a function, used especially for logarithmic and trigonometric functions (i.e. in log(x + 1), x + 1 is the argument).

Arithmetic sequence: A sequence such as 1, 5, 9, 13, 17 or 12, 7, 2, –3, –8, –13, –18, which has a constant difference between consecutive terms called d. ($d = a_n - a_{n-1}$ for sequence 1: 9 - 5 = **4** and 17 – 13 = **4**, hence d = 4 or sequence 2: -3 – 2 = **-5** and -8 – (-3) = **-5** hence d = -5). The first term is a_1, the n^{th} term is a_n.

Asymptote: A line in the plane that is used to help describe the behavior of the graph of a function or a conic section.

- **Horizontal:** Some functions have the property that, as x approaches $\pm\infty$, the graph becomes arbitrarily close to a horizontal line. (A function may intersect its horizontal asymptote.)

- **Vertical:** Some functions have the property that, as x approaches a value c for which the function is undefined, the function values approach $\pm\infty$, making the graph become arbitrarily close to the vertical line x = c. (A function never intersects a vertical asymptote.)

- **Oblique/Slant:** Some functions have the property that, as x approaches $\pm\infty$, the graph becomes arbitrarily close to an oblique line. (A function may intersect its oblique asymptote.)

Augmented matrix: An augmented matrix is formed when the coefficient matrix of a system of linear equations is expanded to include either the column matrix of constants from the same system of equations or the appropriate identity matrix.

Axis of rotation: An axis of rotation is a line about which a figure (or graph) is rotated in space. Assume *P* is a point on the figure (or graph) of distance *d* from the axis of rotation. Then the resulting rotation of the point P is a circle (in space) where all points on the circle are a distance *d* from the axis of rotation.

Axes pertaining to conics:
- **Major:** The line segment through the foci of an ellipse with endpoints on the ellipse.

- **Minor:** The line segment that perpendicularly bisects the major axis of an ellipse with endpoints on the ellipse.
- **Transverse**: The line segment whose endpoints are the vertices of a hyperbola.

- **Conjugate:** The line segment of length $2b$ that is perpendicular to the transverse (focal) axis and has the center of the hyperbola as its midpoint, where the equation of the hyperbola is $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$ or $\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$ and whose center is (h,k).
- **Axis of symmetry**: The axis of symmetry of a two-dimensional figure is a line such that, if a perpendicular is constructed, any two points lying on the perpendicular are at equal distances from the axis of symmetry. The axis of symmetry divides a figure or graph into two figures or graphs that are reflection of each other across the axis.

Binomial coefficients: The coefficients of the terms in the expansion of $(a + b)^n$. The coefficient of the rth term (where the first term of the expanded binomial is the zeroeth term) can be found by calculating C(n, r), the number of combinations of choosing r items out of n items. E.g., in $(a + b)^4 = 1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$, the binomial coefficients are 1, 4, 6, 4, 1. They are symbolized by C(n, r) or ${}_nC_r$ or $\binom{n}{r}$ and may be found using the formula: $\frac{n!}{r!(n-r)!}$.

Binomial Theorem: A formula for the expansion of a binomial raised to any positive integer, n.

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \dots + \binom{n}{n-1}ab^{n-1} + \binom{n}{n}b^n.$$

Bounds for zeros of a polynomial:

- An upper bound for the zeros of a polynomial is a real number, *M*, such that all zeros of the polynomial are less than *M*. When using the Upper Bound Theorem, *M* must be non-negative.
- A lower bound for the zeros of a polynomial is a real number, *m*, such that all zeros of the polynomial are greater than *m*. When using the Lower Bound Theorem, *m* must be a non-positive number.

Center of a conic: The midpoint of the major axis of an ellipse or transverse axis of a hyperbola.

Circular functions: The trigonometric functions are sometimes referred to as circular functions since they may be defined in terms of the amount of arc (length) spanned by an angle on a circle.

Coefficient matrix: The coefficient matrix of the linear system consists of the matrix whose entries are the coefficients of x, y and z written in the same relative positions, i.e. for the system

$$\begin{aligned} x + 2y - 3z &= 4 \\ 3x \qquad + z &= 5 \\ -x - 3y + 4z &= 0 \end{aligned}$$

the coefficient matrix would be $\begin{bmatrix} 1 & 2 & -3 \\ 3 & 0 & 1 \\ -1 & -3 & 4 \end{bmatrix}$.

Common difference: The constant difference between successive terms of an arithmetic sequence ($d = a_{n+1} - a_n$). E.g.,, the common difference of the arithmetic sequence 16, 12, 8, … is -4.

Common ratio: The constant ratio between successive terms of a geometric sequence $r = \frac{a_{n+1}}{a_n}$. E.g., , the common ratio of the geometric sequence $27, 18, 12, 8, \ldots$ is $\frac{2}{3}$.

Completing the square: The process of transforming a quadratic polynomial into the product of the leading coefficient and the square of a binomial, plus a constant. E.g., $y = ax^2 + bx + c$ is equivalent to $y = a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$ The completed square form, sometimes referred to as the *vertex or standard graphing form* , is useful for finding the vertex of a quadratic polynomial and, therefore, useful to find the maximum or minimum value of a quadratic function.

Complex numbers:

- **Rectangular form**: A complex number *z* of the form $z = a + bi$, where *a* and *b* are real numbers and $i = \sqrt{-1}$.
- **Polar (or trigonometric) form**: A complex number *z* of the form $z = |z|(\cos\theta + i\sin\theta)$ or |z|cisθ, where $|z| = \sqrt{a^2 + b^2}$ and θ is the angle formed by the ray from the origin through *z* and the positive real axis in the complex plane (see complex plane definition). The |z| is called the modulus of *z*, and θ is called the argument of *z*.

Complex plane: Complex numbers can be plotted on the *complex plane* where the horizontal axis is the real axis, and the vertical axis is the imaginary axis. E.g., 2 – 3i is plotted as the point (2, -3) in the complex plane.

Composition of functions: If *f* and *g* are functions, then the composition of *f* with *g*, written as $f \circ g$ is another function resulting from using g(x) as an input for f i.e. $(\boldsymbol{f} \circ \boldsymbol{g})(\boldsymbol{x}) = \boldsymbol{f}(\boldsymbol{g}(\boldsymbol{x}))$
This process is similar to the algebraic process of substitution. The domain of the composite function $f \circ g$, consists of all values of *x* in the domain of g such that $g(x)$ is in the domain of *f*.

Conditional Equation: An equation that is true for some value(s) of the variable(s) and not true for others. Example: The equation 2x – 5 = 9 is conditional because it is only true for x = 7. Other values of x do not satisfy the equation.

Conic Sections: A conic section (also called a conic) is a curve obtained by intersecting a double (napped) right circular cone with a plane. The type of conic obtained depends on how the plane intersects the double cone. The general equation of a conic is $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$.

- **Circle:** If the plane is perpendicular to the axis of the cone and does not pass through the apex of the cone, then the conic obtained is a circle. Circles may be considered as special cases of ellipses. $(x – h)^2 + (y – k)^2 = r^2$
- **Parabola:** If the plane is parallel to the side of the cone does not pass through the apex, then the conic obtained is a parabola. $y = a(x – h)^2 + k$ or $x = a(y – k)^2 + h$
- **Hyperbola:** If the plane passes through both the upper and lower nappes of the cone and does not pass through the apex, then the conic obtained is a hyperbola. Therefore, the curve of a hyperbola has two parts. $\frac{(\boldsymbol{x}-\boldsymbol{h})^2}{\boldsymbol{a}^2} - \frac{(\boldsymbol{y}-\boldsymbol{k})^2}{\boldsymbol{b}^2} = 1$ **or** $\frac{(\boldsymbol{y}-\boldsymbol{k})^2}{\boldsymbol{a}^2} - \frac{(\boldsymbol{x}-\boldsymbol{h})^2}{\boldsymbol{b}^2} = 1$
- **Ellipse:** If the plane passes completely through only one nappe of the cone, then the conic obtained is an ellipse. Ellipses are closed curves. $\frac{(\boldsymbol{x}-\boldsymbol{h})^2}{\boldsymbol{a}^2} + \frac{(\boldsymbol{y}-\boldsymbol{k})^2}{\boldsymbol{b}^2} = 1$

- **Degenerate**: When the plane passes through the apex of the cone, one of three possible degenerate cases will occur (a straight line, a point, or a pair of intersecting lines).

Conjugate: The conjugate of a binomial of the form a + b is a - b. For example the conjugate of $2-\sqrt{3}$ is $2+\sqrt{3}$. The conjugate of a complex number a + bi is a – bi. E.g., the conjugate of -1 + 2i is-1 – 2i.

Consistent System: A system of linear equations is consistent if it has at least one solution.

Constant Matrix: The constant matrix of the linear system $\left\{\begin{array}{l} x+2y-3z=4 \\ 3x \qquad +z=5 \\ -x-3y+4z=0 \end{array}\right\}$ consists of the matrix whose entries are the constants 4, 5 and 0 written in the same relative positions, i.e., $\begin{pmatrix} 4 \\ 5 \\ 0 \end{pmatrix}$.

Continuity: Continuity is a property of functions. Roughly speaking, the graph of a continuous function is a single unbroken curve with no breaks, holes or jumps. Continuity at a point *c* is defined in terms of limits, i.e., the function is defined at that point ($f(c)$ exists) and the limit of the function approaches the value $f(c)$ as *x* approaches *c*. A function is continuous everywhere (or simply continuous) if it is continuous for all numbers in its domain.

Convergence of a sequence: Roughly speaking, an infinite sequence converges to some (finite) number *L* if the terms of the sequence get closer and closer to *L*. A sequence $\{a_n\}$ converges to *L*, if the terms of the sequence get closer and closer to L as n goes to infinity.

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Cosecant: The cosecant of an angle is the reciprocal of the sine of the angle.

For a right triangle, the $\csc(\theta)=\dfrac{\textbf{\textit{hypotenuse}}}{\textbf{\textit{opposite side}}}$. For a point (x,y) on the unit circle, the $\csc(\theta)=\dfrac{1}{y}$.

Cosecants are not defined when the $\sin(\theta)=0$.

Cosine: For a right triangle, the $\cos(\theta)=\dfrac{\textbf{\textit{adjacent side}}}{\textbf{\textit{hypotenuse}}}$. For a point (x, y) on the unit circle, the $\cos(\theta)=x$.

Cotangent: The cotangent of an angle is the reciprocal of the tangent of the angle. For a right triangle, the $\cot(\theta)=\dfrac{\textbf{\textit{adjacent side}}}{\textbf{\textit{opposite side}}}$. For a point (x, y) on the unit circle, the $\cot(\theta)=\dfrac{x}{y}$. The cotangent is not defined when the tan(θ) = 0 or the sin(θ) = 0.

Coterminal angle: Two angles are coterminal if when placed in standard form with the initial side as the positive x-axis, they share the same terminal side. The measures of coterminal angles differ by integral multiples of 360^o or 2π radians.

Cramer's Rule: Cramer's Rule is an algorithm for finding the solution of $n \times n$ systems of linear equations using determinants of corresponding square matrices. While not practical for solving large systems, it is important because it gives an explicit expression for the solution of a system.

Decreasing functions: A function f is decreasing if for all x in the domain of f, $f(x_2) < f(x_1)$ whenever $x_2 > x_1$. The graph of a decreasing function moves downward as it is followed from left to right. For example, any line with a negative slope is decreasing.

Degree of a Polynomial: The highest of the degrees of any term in the polynomial.

Degree of a Term: The power or sum of powers in a term. Example: x^3 has degree 3 and x^2y^3 has degree 5.

Degrees (of an Angle): A unit of angle measure equal to $\frac{1}{360}$ of a complete revolution. There are 360 degrees in a circle. Degrees are indicated by the ° symbol, so 35° means 35 degrees.

DeMoivre's Theorem: A formula useful for finding powers and roots of complex numbers written in trigonometric form. Given $z = r\cos(\theta) + i\sin(\theta)$ or $z = r\text{cis}(\theta)$:

$$z^n = r^n\text{cis}(n\theta), \text{ and } \sqrt[n]{z} = \sqrt[n]{r}\,cis\left(\frac{\theta + 2k\pi}{n}\right) \text{ for } k = 0, 1, \ldots, n-1.$$

Dependent system: A system of equations in which at least one of the equations is a linear combination of one or more of the others. If the system is consistent, then it has an infinite number of solutions.

Dependent variable: Formally, a dependent variable is a variable in an expression, equation, or function that has its value determined by the choice of value(s) of other variable(s).

Descartes' Rule of Signs: A method for determining the number of possible positive, negative, and non-real roots of a polynomial.

Determinant: A single number obtained from a matrix that reveals a variety of the matrix's properties. Note: Although the notation for the determinant looks likes an absolute value it is not.

Directrix: A fixed line that, together with a focus, is used to determine the locus of points that form a conic section. For example, a parabola is the locus of points such that the distance to the focus equals the distance to the directrix. The directrix of a parabola is always perpendicular to the axis of symmetry.

Discontinuity: A point at which the graph of a relation or function is not connected. Discontinuities can be classified as either removable or essential. There are several kinds of essential discontinuities, one of which is the step discontinuity.

Discriminant of a quadratic: The number $D = b^2 - 4ac$ determined from the coefficients of the equation $ax^2 + bx + c = 0$. The discriminant reveals the type of roots the equation has. When D is positive, 2 unequal, real roots; when D is negative, 2 unequal complex roots, and when D equals zero, one repeated real solution occurs. Furthermore, if D is a perfect square, 2 unequal, rational roots occur. Note: $b^2 - 4ac$ is the radicand in the quadratic formula.

Divergence of a sequence: A sequence $\{a_1, a_2, \ldots a_n\}$ is said to diverge if the values fail to approach a finite limit as n goes to infinity. Example: {2, 4, 8, 16, …} and {-1, 1, -1, 1, -1,…} both diverge.

Domain: The set of values of the independent variable(s) for which a function or relation is defined. Typically for functions, this is the set of x-values that give rise to real y-values.

Dot product: The number found when the corresponding components of two vectors are multiplied and then summed. For two vectors to have a dot product, they must have the same number of components.

Eccentricity: A nonnegative number that specifies the shape of a conic. The fixed ratio of the distance from a point *P* to a focus compared to the distance from the *P* to the directrix. The eccentricity of a circle is 0 and of a parabola is 1.

Element (of a matrix): See Entry (of a matrix).

Elimination method: A method for solving systems of linear equations involving linear combinations of these equations. The goal of this method is to eliminate at least one of the system's variables and, therefore, to make it easier to solve the system.

End behavior: The behavior of the graph of a function in the coordinate plane as $x \to \pm\infty$,

Entry (of a matrix): Also called an element (of a matrix). Any of the (real) numbers in a matrix. The entries in a matrix are often referred to by the notation $a_{i,j}$ where i indicates the row and j indicates the column in which the entry appears.

Factorization Theorem: $(x-c)$ is a factor of a polynomial if and only if c is a zero of the polynomial.

Foci (*singular* focus): Foci are fixed points that are used to determine the locus of points of a conic. For example, the center of the circle determines a circle of fixed radius *r*. For ellipses, the foci lie on the major axis. For hyperbolas, the foci lie on the extension of the transverse axis.

Function: A function is a relation (set of ordered pairs) in which, for each value of the first component of the ordered pairs, there is *exactly one* value of the second component.

Fundamental Theorem of Algebra: A polynomial function of degree n > 0, has n complex zeros. (Some zeros may be multiple zeros; some may be real).

Gaussian elimination method: An algorithm (prescribed method) for solving a system of n linear equations in n unknowns. Specified elementary row operations reduce a matrix representation of the system to row echelon form. The "last" unknown can be solved directly, and the remaining unknowns can be found by back substitution.

Gauss-Jordan elimination method: An extension of the Gaussian elimination method whereby the matrix representation of the system is transformed to reduced row echelon form. Here, all of the unknowns can be solved directly.

Geometry of a system of linear equations: For a system involving two variables (x and y), each linear equation determines a line on the xy-plane. Because a solution to a linear system must satisfy all of the equations, the solution set is the intersection of these lines, and hence is either a line, a single point, or the empty set. For three variables, each linear equation determines a plane in three-dimensional space, and the solution set is the intersection of these planes. Thus the solution set may be a plane, a line, a single point, or the empty set.

Geometric sequence: A sequence $\{a_n\}$ in which the ratio of consecutive terms is constant. This constant r is called the common ratio of the sequence, and r = $\frac{a_n}{a_{n-1}}$. For example the sequence 2, 6, 18, 54, 162 has a constant ratio of 3 since r = $\frac{6}{2}$ and $\frac{18}{6}$. The first term is a_1, the n^{th} term is a_n, where $a_n = a_1 r^{n-1}$.

Graph (of a function): The set of all points in the coordinate plane corresponding to the ordered pairs $(x, f(x))$ for x in the domain of f.

Half-angle identity: Trigonometric identity involving angles of the form $\frac{\theta}{2}$. These include:

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}} \text{ and } \sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}} \quad \text{ and } \tan\left(\frac{\theta}{2}\right) = \pm\frac{\sin(\theta)}{1-\cos(\theta)}$$

Half-life: The amount of time for half of a radioactive substance to decay, i.e., for its mass to be reduced by one-half.

Horizontal asymptote: The line y = b is a horizontal asymptote of the graph of a function *f* if $f(x) \to b$ as either $x \to \infty$ or $x \to -\infty$. Note graphs may cross horizontal asymptotes.

Horizontal Line Test: A visual test to determine whether a function is 1-to-1. If every horizontal line crosses the graph of a function at most once, then the function is said to be 1 to 1 and it has an inverse. The inverse of a 1 to 1 function is also a 1 to 1 function.

Hyperbola: A conic defined as the locus of points in the coordinate plane such that the difference of the distances from two fixed points (the foci) is a constant. The angle of its cutting plane is greater than the angle of the cone with respect to the horizontal.

Identity: An equation involving variables that is true for all elements in its domain. Compare to conditional equation.

Identity matrix: A square matrix denoted by the letter *I* with ones along the main diagonal and zeros in all other positions. For a 3 X 3 matrix $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Given any other square matrix with the same dimensions as *I*, $IA = AI = A$ for all such *A*.

Inconsistent system: A system of linear equations with no solution.

Increasing function: A function whose values are increasing. The graph of an increasing function moves upward as it is followed from left to right. A function f is increasing if for all x in the domain of f, $f(x_2) > f(x_1)$ whenever $x_2 > x_1$.

Independent system: A system of equations in which no equation is a linear combination of the others.

Independent variable: The variable(s) of a function whose value determines the value of the dependent variable. (E.g., given y = x + 4, x is the independent variable.)

Inverse matrix: The matrix B is the inverse of the square matrix A if BA = I=AB (I is the identity matrix). The inverse matrix for A is denoted A^{-1}.

Invertible matrix: Any square matrix, whose determinant is nonzero, is invertible and hence has an inverse.

Irreducible quadratic: A quadratic expression that cannot be factored over the real number system (E.g.: $x^2 + 4$ is irreducible over the real numbers, however $x^2 - 2 = (x + \sqrt{2})(x - \sqrt{2})$ and is therefore reducible.)

Law of Cosines: Given all three sides of a triangle or two sides and the angle between them (included angle), the remaining parts of the triangle can be found by using one of the following forms for the Law of Cosines:

Sides: $a^2 = b^2 + c^2 - 2bc(\cos\alpha)$; $b^2 = a^2 + c^2 - 2ac(\cos\beta)$; $c^2 = a^2 + b^2 - 2ab(\cos\chi)$

Angles: $(\cos\alpha) = \dfrac{b^2 + c^2 - a^2}{2bc}$; $(\cos\beta) = \dfrac{a^2 + c^2 - b^2}{2ac}$; $(\cos\chi) = \dfrac{a^2 + b^2 - c^2}{2ab}$

NOTE: The angle in the cosine is the angle opposite the missing side.

Limit:

- **of a sequence**: An infinite sequence of numbers a_n, has a limit if the values converge to a finite number L. Technically, the limit of the absolute value of $(a_n - L)$ becomes arbitrarily small when n > N for some positive integer N. An example of an infinite sequence which converges is 1/2, 1/3, 1/4, ..., which converges to (and thus has a limit of) 0.
- **of a function at infinity:** As x approaches infinity the value of the function, f(x), converges to some finite number L Technically, the limit of the absolute value of (f(x) - L) becomes arbitrarily small as x approaches infinity. An example of a limit of a function is f(x) = 1/x which converges to (and thus has a limit of) 0. Functions with limits at infinity have graphs with horizontal asymptotes.

- **of a function at a point:** Assume that f(x) is defined for all x near c (i.e., in some open interval containing c), but not necessarily at c itself. We say that the limit of f(x) as x approaches c is equal to L if the absolute value of (f(x) - L) becomes arbitrarily small when x is any number sufficiently close (but not equal) to c. We also say that f(x) approaches or converges to L as x approaches c. Note that f(x) doesn't need to be defined at c. Even if f(x) is defined at c, f(c) doesn't necessarily need to be equal to the limit L. (If f(c) = L, then f is continuous at c.)

Locus of points: A set of points, connected by a function (rule), that forms a geometric figure or graph. E.g., a circle can be defined as the locus of points in a plane that are all the same distance from a given point.

Lower Bound Theorem: Used to determine if a domain value is a lower bound for the real roots of a polynomial. Given any negative real number, c, and a polynomial with real coefficients, P(x), then there are no real roots less than c if when P(x) is divided by (x – c) the coefficients of the resulting polynomial alternate in sign.

(E.g., Given $P(x) = x^3 - 4$ and c = -2, $P(x) \div (x - (-2)) = x^2 - 2x + 4 - \dfrac{4}{\boldsymbol{x}+2}$. The coefficients are +1, -2, +4, and -4 and hence alternate.) NOTE: This only indicates that the value c is a lower bound, not necessarily the greatest lower bound. A 0 can be treated as either positive or negative.

Magnitude (of a vector): The length of a vector. Given $v = ai + bj$, the magnitude of vector v is $\|v\| = \sqrt{a^2 + b^2}$.

Mathematical induction: A method of mathematical proof typically used to establish that a given statement is true for an infinite sequence of natural numbers. It is done by proving that the first statement in the infinite sequence of statements is true, and then proving that if any one statement in the infinite sequence of statements is true, then so is the next one.

Matrix: A rectangular array of numbers generally represented by capital letters. Matrices are written using brackets. If m and n are positive integers, then an m by n matrix is a rectangular array in which each entry, aij, of the matrix is a number. An mxn matrix has m rows and n columns.

Ex. M = $\begin{bmatrix} 2 & 3 \\ -1 & 4 \\ 0 & -2 \end{bmatrix}$ is a 3 by 2 matrix where the entry $a_{2,1} = -1$.

- **Minor: The minor, M_{ij},** is the (n-1) by (n-1) sub matrix obtained from deleting the ith row and the jth column. Hence for the matrix M = $\begin{bmatrix} 1 & 2 & 3 \\ -2 & 0 & -1 \\ 4 & -3 & 1 \end{bmatrix}$, the minor $M_{2,3} = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$
- **Cofactor:** The product of the determinant of a minor of a square matrix times +/- 1 depending on the position in the original matrix. It is used in calculating the determinant of the original square matrix. Let M_{ij} be the minor for element a_{ij} in an $n \times n$ matrix. The cofactor of a_{ij}, written A_{ij}, is $A_{ij} = (-1)^{i-1}|Mij|$.

Matrix addition: Matrices are added by adding the elements in corresponding positions. E.g., $A+B=\left[a_{ij}+b_{ij}\right]$.

Matrix dimensions: A matrix, A, is said to have dimensions m x n, where m is the numbers of rows and n is the number of columns in the matrix.

Matrix element: One of the entries in a matrix. The address of an element is given by listing the row number then the column number. E.g., a_{ij}, is the element in row i and column j.

Matrix equation: Linear systems may be represented by a matrix equation in the form $AX=B$, where A is the coefficient matrix, X is the variable matrix and B is the constant matrix. The solution is $X=A^{-1}B$ where A^{-1} is the inverse matrix if the inverse matrix exists.

Matrix multiplication: Matrices may not be multiplied unless their dimensions are compatible, that is, in AB, the numbers of columns of matrix A must equal the number of rows of matrix B. Note: When an *m x n* matrix is multiplied by an *n x r* matrix, the product has dimensions *m x r*. Each element of the product matrix is the sum of the products of each row element with its corresponding column element.

Example: A = $\begin{bmatrix} 2 & 3 \\ -1 & 4 \\ 0 & -2 \end{bmatrix}$ and let B = $\begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$ then AB is the 3 by 2 matrix

$$\begin{bmatrix} 2\cdot 1+3\cdot 4 & 2\cdot 2+3\cdot(-3) \\ -1\cdot 1+4\cdot 4 & -1\cdot 2+4\cdot(-3) \\ 0\cdot 1+(-2)\cdot 4 & 0\cdot 4+(-2)\cdot(-3) \end{bmatrix} = \begin{bmatrix} 14 & -5 \\ 15 & -14 \\ -8 & 6 \end{bmatrix}$$

Matrix row operations: Operations that are used to transform a matrix into a different matrix while not changing the solution set of the system of linear equations represented by the matrix. A specific use of matrix row operations would be Gaussian Elimination.

These operations are:

1. Any row in a matrix may be interchanged with any other row.
2. Any row in a matrix may be multiplied by a nonzero scalar.
3. Any row in a matrix may be replaced by the sum of that row and a multiple of any other row.

Modulus of a complex number: The distance between a complex number and the origin on the complex plane. The modulus or absolute value of a + bi is written |a + bi|, and the formula for |a + bi| is $\sqrt{a^2+b^2}$. For a complex number in polar form r(cos θ + isin θ), the modulus is r.

Oblique asymptote: See Asymptote.

Oblique line: Any line with a positive or negative slope.

One to One: A function is one to one if for every range (output) value there is a only one domain (input) value.

Parametric Equations: A method for defining a plane curve using functions with a common parameter. A parameter is a variable in terms of which two or more variables are defined. E.g., to define a circle of radius A, we could define x(t) = Acos(t) and y(t) = Asin(t) where t is an angle of radian measure. As t increases from 0 to 2π, the graph of a circle is drawn counter-clockwise.

Perpendicular (orthogonal) vectors: At a 90° angle. Note: Perpendicular lines have slopes that are negative reciprocals. (In n-space, orthogonal vectors have a dot product equal to zero.)

Point of symmetry: P is a point of symmetry for a graph if the graph can be rotated 180^o about P and the resulting image is identical to the original graph.

Polar coordinates: A way to describe the location of a point on a plane. A point is represented by the coordinates (r,θ), where r is the distance from the point to the origin and θ is the angle measured counterclockwise from the polar (positive x) axis to the segment connecting the point to the origin. Note: With polar coordinates a given point has many possible representations. θ has many possible values depending on which coterminal angle is chosen, and r can be positive or negative.

Poles of a rational function: The zeros of the denominator which result in a hole in the graph or the vertical asymptotes in the graph of a rational function.

Polynomials: A sum of monomials, where a monomial is the product of a number (called a coefficient) and a non-negative integer power of a variable. A polynomial in one variable is given by $a_nx^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$. The degree of this general polynomial is n. Examples of polynomials are 3, -2x, and $2x^3 - x + 5$.

Pythagorean Theorem: An equation relating the lengths of the sides of a right triangle. The sum of the squares of the legs of a right triangle is equal to the square of the hypotenuse.

Pythagorean Identities: Trigonometric identities relating sine with cosine, tangent with secant, and cotangent with cosecant. Derived from the Pythagorean Theorem. These are: $\sin^2(x) + \cos^2(x) = 1$; $\cot^2(x) + 1 = \csc^2(x)$; $1 + \tan^2(x) = \sec^2(x)$.

Quadratic equation: An equation that can be written in the form $ax^2 + bx + c = 0$ where a, b, and c are all real numbers and $a \neq 0$.

Quadratic formula: A formula for finding the roots of a quadratic equation. Given an equation in the form $ax^2 + bx + c = 0$, the roots are given by: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Radians: A unit for measuring angles. 180° = π radians, and 360° = 2π radians. The number of radians in an angle equals the number of radii it takes to measure a circular arc intercepted by that angle. .

Radius: A line segment connecting the center of a circle or sphere with a point on the circle or sphere. The word radius also refers to the length of this segment.

Range: The set of y-values of a function or relation. More generally, the range is the set of values assumed by a function or relation over all permitted values of the independent variable(s).

Rational zero (root) theorem: A theorem that provides a complete list of possible rational roots of the polynomial equation $a_nx^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0 = 0$ where all coefficients are integers.

This list consists of all possible numbers of the form $\pm\frac{c}{d}$, where c and d are integers. c must be a factor of a_0. d must be a factor of the leading coefficient a_n.

Rationalize the denominator: The process of finding an equivalent fraction (or rational expression) such that no radicals remain in the denominator.

Rectangular coordinates: Points in the plane or in space defined by signed distances from the origin along perpendicular axes. Also called Cartesian coordinates and usually represented in the form (x, y) or (x, y, z).

Reduced row echelon matrix: See row echelon matrix.

Right triangle: A triangle that has a right (90°) interior angle.

Roots (zeros) of an equation: A solution to an equation of the form f(x) = 0. Roots may be real or complex. Note: The roots of f(x) = 0 are the same as the zeros of the function f(x). Sometimes in casual usage the words root and zero are used interchangeably.

Rotation: A transformation in which a plane figure turns around a fixed center point. In other words, one point on the plane, the center of rotation, is fixed and everything else on the plane rotates about that point by a given angle.

Row echelon matrix (reduced row echelon matrix): A matrix form used in solving linear systems of equations. An n *x* r matrix contains ones along a diagonal starting in row 1 column 1 to row n column n. Zeros fill in the element positions below the ones. (For a reduced row echelon matrix, zeros also fill in the element positions above the ones.) Systems of linear equations can be written as augmented matrices. If the determinant of the coefficient matrix is not zero, then the augmented matrix can be transformed into row echelon form (reduced row echelon form) by elementary row operations. In row echelon form the "last" variable can be solved directly, and the remaining variables can be solved by back-substitution. (In reduced row echelon form, each variable can be solved directly.)

Scalar: Any real number, or any quantity that can be measured using a single real number. Temperature, length, and mass are all scalars. A scalar is said to have magnitude but no direction. A quantity with both direction and magnitude, such as force or velocity, is called a vector. Scalar can also refer to a single number when distinguishing it from a matrix.

Scalar (dot) product (of two vectors): In two dimensions, (ai + bj)•(ci + dj) = ac + bd. In three dimensions, (ai + bj + ck)•(di + ej + fk) = ad + be + cf. In either case, u • v = |u| |v| cos θ, where θ is the angle between the vectors.

Scalar matrix: A scalar multiple of an identity matrix.

Scalar multiplication (of a matrix or vector): The product of a scalar with a matrix or vector.

Sequence: A ordered list of numbers set apart by commas, such as 1, 3, 5, 7, ... Each number is called a term of the sequence. The list may consist of a finite or an infinite number of terms. (see Arithmetic and Geometric Sequences.)

Series: The sum of the terms of a sequence. Example:The series for the sequence 1, 3, 5, 7, 9, ... , 131, 133 is the sum 1 + 3 + 5 + 7 + 9 + ... + 131 + 133.

Sigma notation: A notation using the Greek letter sigma (Σ) that allows a long sum to be written compactly. E.g., 1 + 2 + … + 100 = $\sum_{1=1}^{100} i$

Sine: For a right triangle, the $\sin(\vartheta) = \frac{\textbf{\textit{opposite side}}}{\textbf{\textit{hypotenuse}}}$. For a point (x, y) on the unit circle, the

sin(θ) = y.

System of linear equations: A system of equations in which each equation is linear. For any linear system, exactly one of the following will be true: There is only one solution, there are infinitely many solutions (under determined), or there are no solutions (over determined or inconsistent).

Square matrix: A matrix with equal numbers of rows and columns.

Substitution method: A method used to eliminate one of the variables by replacement when solving a system of equations. .

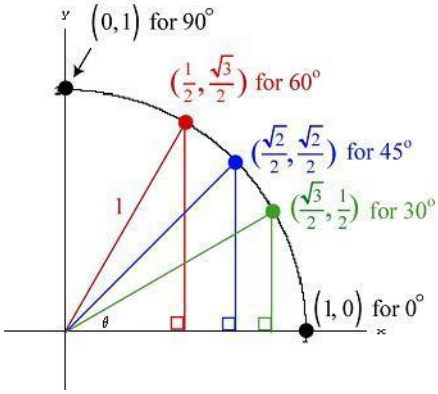
Tangent: For a right triangle, the $\tan(\theta) = \frac{\textbf{\textit{opposite side}}}{\textbf{\textit{adjacent side}}}$. For a point (x, y) on the unit circle, the

$\tan(\theta) = \frac{\boldsymbol{y}}{\boldsymbol{x}}$. The tangent is not defined when the cos(θ) = 0.

Translation: A transformation in which a graph or geometric figure is moved to another location without any change in size or orientation.

Unit vector: A vector of unit length. Often a unit vector is written using the ^ symbol. E.g., û is a unit vector pointing in the same direction as the vector **u**.

Unit Circle: A circle of radius 1 centered at the origin. An angle, theta q, is measured in a counter-clockwise direction with its vertex at the origin and its initial side lying along the positive x-axis. The angle's terminal side is formed by the line segment from the origin to the point (x, y) on the circle. The diagram shows the angles in the first quadrant. Note that the cosine of the angle is the x-coordinate and the sine of the angle is the y-coordinate for the point of intersection of the unit circle and the terminal side of the angle.



Upper Bound Theorem: Used to determine if a domain value is an upper bound for the real roots of a polynomial. Given any positive real number, c, and a polynomial with real coefficients, P(x), then there are no real roots greater than c if when P(x) is divided by (x – c) the coefficients of the resulting polynomial are all positive. (E.g., given $P(x) = x^3 - 4$ and c = 2, $P(x) \div (x - 2) = x^2 + 2x + 4 + \frac{4}{x+2}$. The coefficients are +1, +2, +4, +4 and hence all positive.) NOTE: This only indicates that the value c is an upper bound, not necessarily the lowest upper bound. A 0 can be treated as either positive or negative.

Vector: A directed line segment that has magnitude and direction.

Vector addition: Symbolically, if $\mathbf{v}_1$= ai +bj and $\mathbf{v}_2$= ci +dj, then $\mathbf{v}_1 + \mathbf{v}_2$= (a+c)i + (b+d)j. Graphically, the sum of vectors can be represented by its resultant. If the initial point of $\mathbf{v}_2$ is connected to the terminal point of $\mathbf{v}_1$, then the vector that connects the initial point of $\mathbf{v}_1$ to the terminal point of $\mathbf{v}_2$ is the resultant.

Vertex of a conic section (vertices): For a parabola, the vertex is the turning point. For an ellipse, the vertices are the end points of the major and minor axes. For a hyperbola, the vertices are the end points of the transverse axis.

Vertex of a geometric figure (vertices): For a polygon, vertices are the points where adjacent sides meet. For a polyhedron, vertices are the points where adjacent edges meet. For an angle, the vertex is where the two rays making up the angle originate.

Zeros of a function: Elements of the domain for which the function equals 0. A zero may be a real or complex number. See roots of an equation.